

# On primordial equation of state transitions

---

**Aditya Aravind, Dustin Lorshbough and Sonia Paban**

<sup>1</sup>*Department of Physics and Texas Cosmology Center  
The University of Texas at Austin, TX 78712.*

*E-mail:* [Aditya.phy@utexas.edu](mailto:Aditya.phy@utexas.edu), [Lorsh@utexas.edu](mailto:Lorsh@utexas.edu),  
[Paban@zipppy.ph.utexas.edu](mailto:Paban@zipppy.ph.utexas.edu)

**ABSTRACT:** We revisit the physics of transitions from a general equation of state parameter to the final stage of slow-roll inflation. We show that it is unlikely for the modes comprising the cosmic microwave background to contain imprints from a pre-inflationary equation of state transition and still be consistent with observations. We accomplish this by considering observational consistency bounds on the amplitude of excitations resulting from such a transition. As a result, the physics which initially led to inflation likely cannot be probed with observations of the cosmic microwave background. Furthermore, we show that it is unlikely that equation of state transitions may explain the observed low multipole power suppression anomaly.

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Single Field Inflation</b>	<b>2</b>
2.1	Overview	2
2.2	Observables for General Bogoliubov Parameters	4
2.3	Bounds on Excitation Amplitude	4
2.3.1	Bounds from Backreaction Considerations	4
2.3.2	Special Case: Only $l \lesssim 30$ Modes Excited	6
<b>3</b>	<b>Excitation Mechanism: Instant Transition</b>	<b>6</b>
3.1	Matching Conditions	6
3.2	Observables: Allowed Parameter Space	8
3.3	Comparison with Previous Work	11
<b>4</b>	<b>Excitation Mechanism: Gradual Transition</b>	<b>12</b>
4.1	Transition Model	12
4.2	Observables: Allowed Parameter Space	12
<b>5</b>	<b>Implications for Low Multipole Scalar Power Spectrum Suppression</b>	<b>15</b>
<b>6</b>	<b>Conclusions</b>	<b>15</b>

---

## 1 Introduction

The paradigm of inflation provides a suitable framework for understanding the observed spectrum of cosmological perturbations [1]. Many questions remain regarding the origin of inflation [2–4] and its duration [5, 6]. It has been proposed that the transition to inflation may explain observed anomalies in the cosmic microwave background (CMB) if the duration of inflation is not too long [7].

This paper revisits the generation of spectrum excitations due to the transition from an arbitrary equation of state parameter ( $w$ ) to inflation. We emphasize the importance of using the proper matching conditions across the transition and show that previous studies have drawn incorrect conclusions when using improper matching conditions. By combining observational consistency bounds on the excitation amplitude with the

proper matching conditions we show that the cosmic microwave background likely does not contain imprints from the pre-inflationary universe. Our study emphasizes three points:

1. Observation strongly bounds the amplitude of excitation.
2. The fractional change in  $w$  must be small if a  $w$  transition is observed.
3. It is unlikely that transitions explain the  $(20 \lesssim l \lesssim 30)$  anomaly.

The first point has been made before [8–11], but a more general formulation of the bound is presented in section 2. In section 3 we analytically compute the excited spectrum for the case of an instant transition, allowing us to bound the pre-transition equation of state parameter. In section 4 we discuss the excited spectrum for the case of a gradual transition which is modeled by a hyperbolic tangent function. In section 5 we address the low power anomaly occurring for multipoles  $20 \lesssim l \lesssim 30$  and show that the type of models we discuss cannot explain the anomaly. In section 6, we conclude.

## 2 Single Field Inflation

### 2.1 Overview

In this section we will review the basic equations of inflationary theory, emphasizing the appearance of  $\epsilon$  terms which will play an important role in studying the enhanced spectrum that results from equation of state transitions. The simplest theory for inflation that is consistent with observational data is a minimally coupled scalar field with an FRW metric [12],

$$S = \int \sqrt{-g} \left[ \frac{1}{2} M_P^2 R - \frac{1}{2} (\partial\phi)^2 - V \right], \quad ds^2 = -dt^2 + a^2 dx^2. \quad (2.1)$$

The Einstein field equations may be combined to obtain an expression relating the rate of Hubble parameter change directly to the sum of the energy density and the pressure of the contents in the universe,

$$2M_P^2 \dot{H} = -(\rho + P). \quad (2.2)$$

For a single scalar field with negligible gradients we may write  $\rho = K + V$  and  $P = K - V$ . It is convenient to introduce three dimensionless parameters,

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \quad \epsilon_\phi = 3\frac{K_\phi}{\rho} \text{ and } w = \frac{P}{\rho} = -1 + \frac{2}{3}\epsilon. \quad (2.3)$$

It is clear from (2.2) that  $\epsilon_H = \epsilon_\phi$ . In order to obtain an accelerating geometry we require that the parameter  $\epsilon_H$  be smaller than unity,

$$\ddot{a} = aH^2(1 - \epsilon_H) > 0 \text{ if } \epsilon_H < 1. \quad (2.4)$$

Cosmological observables are obtained by computing correlation functions of gauge invariant fluctuations. The action for the comoving curvature perturbation is given by

$$S_{\mathcal{R}} = -M_P^2 \int \sqrt{-g} \epsilon (\partial \mathcal{R})^2. \quad (2.5)$$

The fields may be Fourier decomposed as

$$\hat{\mathcal{R}} = \int \frac{d^3 k}{(2\pi)^3} \hat{\mathcal{R}}_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}, \quad \hat{\mathcal{R}}_{\vec{k}} = \mathcal{R}_k \hat{a}_k^\dagger + \mathcal{R}_k^* \hat{a}_{-\vec{k}}, \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}'). \quad (2.6)$$

The resulting equation of motion for the mode function is<sup>1</sup>

$$\ddot{\mathcal{R}}_k + \left(3H + \frac{\dot{\epsilon}}{\epsilon}\right) \dot{\mathcal{R}}_k + \frac{k^2}{a^2} \mathcal{R}_k = 0. \quad (2.7)$$

For the case of quasi-de Sitter expansion ( $w \approx -1$ ,  $\epsilon \ll 1$ ), the solution of lowest energy density is given by the Bunch-Davies solution

$$\mathcal{R}_{k,BD} = \frac{1}{\sqrt{2} \epsilon} \frac{H}{M_P \sqrt{2} k^3} \left(1 + i \frac{k}{a H}\right) e^{-ik/aH}. \quad (2.8)$$

The scalar power spectrum has been measured to great accuracy, while the scalar bispectrum and tensor power spectrum have not yet been detected. The scalar power spectrum is typically parameterized by an amplitude  $A_S$  and a scale dependence  $n_s$ ,

$$\langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P_{\mathcal{R}}(k), \quad \Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_S \left(\frac{k}{k_*}\right)^{n_s-1+\mathcal{O}(dn_s/dk)}. \quad (2.9)$$

The observationally obtained values for  $A_S$  and  $n_s$  are given in Table 1.

Scalar Power Spectrum Amplitude	Scalar Power Spectrum Tilt
$\ln(10^{10} A_S) = 3.094 \pm 0.034 (1\sigma)$	$n_s = 0.9645 \pm 0.0049 (1\sigma)$

**Table 1.** Cosmological parameters as measured by Planck TT+TE+EE+lowP [1, 13–15]. The pivot scale used by the Planck experiment is  $k_* = 0.05 \text{ Mpc}^{-1}$ .

The predictions for the Bunch-Davies mode functions are

$$A_S = \frac{1}{8\pi^2 \epsilon} \frac{H^2}{M_P^2}, \quad n_s = 1 + 2\eta - 4\epsilon, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad (2.10)$$

<sup>1</sup>The absence of a friction term proportional to  $\dot{\epsilon}$  in the equation of motion for the tensor mode functions is the reason tensor modes are not enhanced for equation of state transitions in the way that scalars are enhanced.

## 2.2 Observables for General Bogoliubov Parameters

The Bunch-Davies state is the solution of lowest energy density in quasi-de Sitter expansion. However, in general the solution does not need to be the solution of lowest energy density. We write excited solutions as a Bogoliubov transformation of the Bunch-Davies solution

$$\mathcal{R}_{k,\text{excited}} = \alpha_k \mathcal{R}_{k,BD} + \beta_k \mathcal{R}_{k,BD}^* \text{ and } \hat{a}_{\vec{k},\text{excited}}^\dagger = \alpha_k^* \hat{a}_{\vec{k},BD}^\dagger - \beta_k^* \hat{a}_{-\vec{k},BD}. \quad (2.11)$$

Satisfying the canonical commutation relations requires that  $|\alpha_k|^2 - |\beta_k|^2 = 1$ .

Excited states change the cosmological parameters. In terms of the expressions corresponding to the Bunch-Davies solution previously discussed, the new parameter expressions are given by

$$\Delta_{\mathcal{R},\text{excited}}^2 = \Delta_{\mathcal{R},BD}^2 |\alpha_k + \beta_k|^2, \quad A_S = A_{S,BD} |\alpha_{k_*} + \beta_{k_*}|^2, \quad n_s = n_{s,BD} + \frac{d \ln |\alpha_k + \beta_k|^2}{d \ln k}. \quad (2.12)$$

## 2.3 Bounds on Excitation Amplitude

We present the bounds arising from backreaction considerations for different functional forms of Bogoliubov excitations, extending the results of [9–11] to other functional forms, to show that the limits are similar. Bounds that arise from measurements of  $n_s$  and the observational limits on  $f_{\text{NL}}^{\text{loc}}$  turn out to be weaker than the bounds that are obtained from backreaction considerations.

### 2.3.1 Bounds from Backreaction Considerations

The scalar power spectrum has been measured to deviate only slightly from scale invariance. This implies that the modes comprising the observable cosmic microwave background should not vary dramatically in amplitude across the approximately 3-4 decades of modes we observe today. Therefore the modes we observe today should either be excited modes or Bunch-Davies modes. We will compute the bounds on excitation parameters implied by this.

In order for all of the modes comprising the CMB to have exited the horizon during inflation, the highest- $l$  mode must be at least  $n \approx 3 - 4$  [16–18] decades shorter wavelength than the horizon size at the beginning of inflation. To obtain the most conservative bound from backreaction considerations we will assume that modes with higher momentum than the observable CMB modes are not excited. The highest momentum excited mode therefore satisfies

$$p_{\text{UV}} \geq 10^n H. \quad (2.13)$$

The energy density stored in the fluctuations after adiabatic subtraction is given by

$$\langle \rho_{\mathcal{R}} \rangle = M_P^2 \epsilon \int_0^\infty \frac{d^3 k}{(2\pi)^3} \left[ |\dot{\mathcal{R}}_{k,\text{excited}}|^2 - |\dot{\mathcal{R}}_{k,BD}|^2 + \frac{k^2}{a^2} (|\mathcal{R}_{k,\text{excited}}|^2 - |\mathcal{R}_{k,BD}|^2) \right], \quad (2.14)$$

We will consider different functional forms of scale dependence for the Bogoliubov parameter  $\beta_k = |\beta| f_k$  shown in Table 2.

$f_k$	$\langle \rho_{\mathcal{R}} \rangle$
$\Theta(k_{\text{UV}} - k)$	$\frac{ \beta ^2}{8\pi^2} (p_{\text{UV}}^4 + H^2 p_{\text{UV}}^2)$
$\exp[-k/k_{\text{UV}}]$	$\frac{ \beta ^2}{16\pi^2} (3p_{\text{UV}}^4 + H^2 p_{\text{UV}}^2)$
$\exp[-(k/k_{\text{UV}})^2]$	$\frac{ \beta ^2}{16\pi^2} (p_{\text{UV}}^4 + H^2 p_{\text{UV}}^2)$

**Table 2.** The energy density for the different functional forms of  $\beta_k = |\beta| f_k$  that are considered. The first form is that used in [8–11]. As emphasized in [19], the oscillations resulting from instant transitions are very rapid so the effective  $\beta_k$  seen by experiments would appear nearly scale invariant. Bounds on the gaussian form were considered in [20–23]. For a discussion on coherent states see [24, 25].

The energy density stored in the fluctuations should remain sub-dominant to the kinetic energy of the inflaton field,  $\epsilon M_P^2 H^2$ , in order for  $\epsilon_H = \epsilon_\phi$  to remain true. This backreaction bound may be written as

$$\langle \rho_{\mathcal{R}} \rangle < \epsilon M_P^2 H^2 = \frac{H^4}{8\pi^2 A_S} |\alpha_{k_*} + \beta_{k_*}|^2. \quad (2.15)$$

It is convenient to introduce a parameter which is the coefficient ratio for the leading energy density terms in Table 2,

$$c_f = \begin{cases} 1, & f_k = \Theta(k_{\text{UV}} - k) \\ 2/3, & f_k = \exp[-k/k_{\text{UV}}] \\ 2, & f_k = \exp[-(k/k_{\text{UV}})^2] \end{cases}. \quad (2.16)$$

The backreaction bound gives us,

$$p_{\text{UV}} \lesssim \frac{H |\alpha_{k_*} + \beta_{k_*}|^{1/2}}{A_S^{1/4} |\beta|^{1/2}} \times c_f^{1/4} \quad (2.17)$$

Recalling that  $|\alpha_k|^2 = 1 + |\beta_k|^2$ , using (2.13) and (2.17) we obtain an upper bound for  $|\beta|$  given by

$$|\beta| \lesssim 10^{-2n} A_S^{-1/2} c_f^{1/2}. \quad (2.18)$$

We provide the numerical evaluation of the  $|\beta|$  upper bound and the corresponding bounds on  $|\alpha_k + \beta_k|$  in Table 3.

$f_k$	$ \beta $ UB	$ \alpha_{k_*} + \beta_{k_*} $ LB	$ \alpha_{k_*} + \beta_{k_*} $ UB
$\Theta(k_{\text{UV}} - k)$	0.022	0.97	1.022
$\exp[-k/k_{\text{UV}}]$	0.018	0.98	1.02
$\exp[-(k/k_{\text{UV}})^2]$	0.031	0.97	1.03

**Table 3.** Bounds on  $\beta_k = |\beta|f_k$  and  $|\alpha_k + \beta_k|$  arising from backreaction considerations. We have taken  $n = 3$  and  $k_* = 0.05 \text{ Mpc}^{-1} \sim 0.5k_{\text{UV}}$  for the numerical estimates. To obtain the lower bound we have allowed  $\beta_k$  to be negative.

### 2.3.2 Special Case: Only $l \lesssim 30$ Modes Excited

For the lowest  $l$  cosmic variance limited modes ( $l \lesssim 30$ ), one may not use scale invariance to bound the excitation amplitude. Instead, we compare the theoretical prediction with the observational error bar width [13] to obtain a conservative estimate of  $0.5 \lesssim |\alpha_k + \beta_k|^2 \lesssim 2$ .

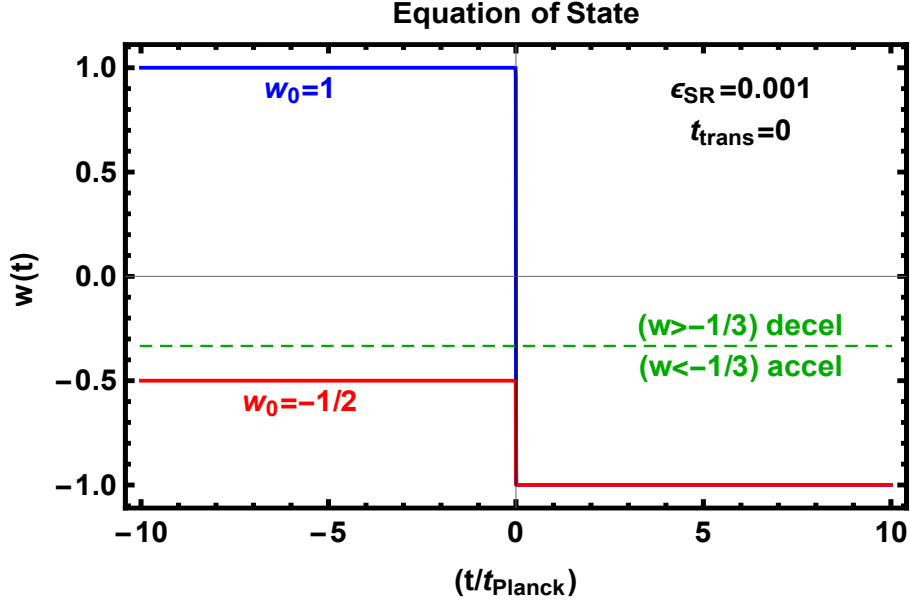
## 3 Excitation Mechanism: Instant Transition

Although unrealistic, the idealized case of an instantaneous transition from one equation of state parameter value to a different value is useful since it allows for an analytical calculation of the excited spectrum. In the next section, the more realistic case of a gradual transition will be discussed. Figure 1 illustrates the transition for several different values of  $w_0$ .

### 3.1 Matching Conditions

The instant transitions that we are considering effectively have a discontinuity in the slow roll parameter  $\epsilon$ . Therefore, one must be careful when determining which quantities related to  $\mathcal{R}$  are continuous since the evolution of  $\mathcal{R}$  depends explicitly on  $\dot{\epsilon}$  according to (2.7). It was emphasized in [19, 26] that the proper matching conditions are given by

$$[\mathcal{R}_k]_{\pm} = 0, \quad [\epsilon \dot{\mathcal{R}}_k]_{\pm} = 0, \quad (3.1)$$



**Figure 1.** Examples of instantaneous transitions. Though not explicitly plotted, we also have studied the transition which increases the equation of state parameter (see Table 5).

where we have used the notation that  $[\cdots]_{\pm}$  denotes the change in a quantity across the transition. The origin of these conditions may be easily seen from the differential equation for scalar fluctuations (2.7), which can be rewritten as

$$\frac{d}{dt} \left( a^3 \epsilon \dot{\mathcal{R}}_k \right) = -k^2 a \epsilon \mathcal{R}_k. \quad (3.2)$$

The fluctuation mode function itself,  $\mathcal{R}_k$ , is continuous. If we note that  $\epsilon = d(1/H)/dt$ , we may time integrate both sides of the equation close to the transition to obtain the continuity condition on the derivative of the mode function,

$$\left[ a^3 \epsilon \dot{\mathcal{R}}_k \right]_{\pm} = \lim_{\delta \rightarrow 0} \int_{(\Delta t)_t = -\delta}^{(\Delta t)_t = \delta} dt \left( -k^2 a \epsilon \mathcal{R}_k \right) = \left[ -k^2 \frac{a}{H} \mathcal{R}_k \right]_{\pm} = 0. \quad (3.3)$$

We have introduced the notation  $(\Delta t)_t = (t - t_{\text{trans}})$  to denote that time difference between the cosmic time and the time of transition.

Microscopically, the inflaton field will take on a uniform value<sup>2</sup> at the transition time  $(\Delta t)_t = 0$ . This implies in the language of [19, 26] that the transition is characterized by a spacetime hypersurface of constant field value, which directly yields (3.1). This is in contrast to the examples studied in [26] in which the transition was characterized by a surface of constant energy density and hence the uniform density gauge

<sup>2</sup>Note that  $\delta\phi = 0$  in the comoving gauge [12].



is more appropriate since  $\delta\rho = 0$  in that gauge. The continuity conditions (3.1) have been verified numerically by time evolving (2.7).

### 3.2 Observables: Allowed Parameter Space

Based on the matching conditions previously discussed, we would like to solve the following system of equations:

$$\begin{aligned} (\Delta t)_t = 0 : \quad \alpha_{0,k} \mathcal{R}_{0,k} + \beta_{0,k} \mathcal{R}_{0,k}^* &= \alpha_{f,k} \mathcal{R}_{\text{SR},k} + \beta_{f,k} \mathcal{R}_{\text{SR},k}^*, \\ (\Delta t)_t = 0 : \epsilon_0 \left( \alpha_{0,k} \dot{\mathcal{R}}_{0,k} + \beta_{0,k} \dot{\mathcal{R}}_{0,k}^* \right) &= \epsilon_{\text{SR}} \left( \alpha_{f,k} \dot{\mathcal{R}}_{\text{SR},k} + \beta_{f,k} \dot{\mathcal{R}}_{\text{SR},k}^* \right). \end{aligned} \quad (3.4)$$

The explicit forms of the functions  $\mathcal{R}_{0,k}$  and  $\mathcal{R}_{\text{SR},k}$  are given in (3.8). The coefficients  $\{\alpha_{0,k}, \beta_{0,k}\}$  account for the fact that the spectrum may be excited prior to the transition to slow-roll inflation. The case of the lowest energy density vacuum state transitioning to inflation is given by the choice  $\alpha_{0,k} = 1$  and  $\beta_{0,k} = 0$ .

In order to solve (3.4), we must find a solution to scalar fluctuation mode equation which is properly normalized. The normalization condition is given by the canonical commutation relation, which may be rewritten as a condition on the Wronskian of the scalar fluctuation mode function as follows

$$2a^3 M_P^2 \epsilon \left( \dot{\mathcal{R}}_k \mathcal{R}_k^* - \mathcal{R}_k \dot{\mathcal{R}}_k^* \right) = i. \quad (3.5)$$

The background geometry evolution is given by

$$\epsilon = \begin{cases} \epsilon_{\text{SR}} & (\Delta t)_t < 0 \\ \epsilon_0 & (\Delta t)_t > 0 \end{cases}, \quad H(t) = H_t [1 + \epsilon H_t (\Delta t)_t]^{-1}, \quad a(t) = a_t \left( \frac{H_t}{H(t)} \right)^{1/\epsilon}. \quad (3.6)$$

We have defined  $H_t$  and  $a_t$  as the Hubble parameter and scale factor at the time of transition,  $(\Delta t)_t = 0$ . It is convenient to introduce the variables

$$\begin{aligned} (\Delta t)_t < 0 : \quad \tilde{\tau}_0 &= \int \frac{dt}{a} = \frac{1}{(-1 + \epsilon_0)} \frac{1}{a(t)H(t)}, \\ (\Delta t)_t > 0 : \quad \tilde{\tau}_{\text{SR}} &= \int \frac{dt}{a} = \frac{1}{(-1 + \epsilon_{\text{SR}})} \frac{1}{a(t)H(t)}. \end{aligned} \quad (3.7)$$

The properly normalized solution to the scalar equation of motion (2.7) for a constant

equation of state is given as<sup>3</sup>

$$\begin{aligned}
(\Delta t)_t < 0 : \quad \mathcal{R}_{0,k}(t) &= \sqrt{\frac{\pi}{2^3 \epsilon_0}} \frac{1}{M_P a(t)} \sqrt{|\tilde{\tau}_0|} H_{\nu_0}^2 [k|\tilde{\tau}_0|], \quad \nu_0 = \frac{3(1-w_0)}{2(1+3w_0)} \\
(\Delta t)_t > 0 : \quad \mathcal{R}_{\text{SR},k}(t) &= \sqrt{\frac{\pi}{2^3 \epsilon_{\text{SR}}}} \frac{1}{M_P a(t)} \sqrt{|\tilde{\tau}_{\text{SR}}|} H_{\nu_{\text{SR}}}^2 [k|\tilde{\tau}_{\text{SR}}|], \quad \nu_{\text{SR}} = \frac{3(1-w_{\text{SR}})}{2(1+3w_{\text{SR}})}
\end{aligned} \tag{3.8}$$

Employing the matching conditions (3.4) we are able to write the Bogoliubov parameters for an arbitrary choice of  $w_0$  and initial excitation parameters  $\alpha_0$  and  $\beta_0$  as

$$\alpha_{f,k} = \sqrt{\frac{\epsilon_0}{\epsilon_{\text{SR}}}} \sqrt{\left| \frac{\epsilon_{\text{SR}} - 1}{\epsilon_0 - 1} \right|} \frac{\alpha_{\text{num}}}{\alpha_{\text{denom}}}, \quad \beta_{f,k} = \alpha_{f,k} \text{ with } H_{f(\nu_{\text{SR}})}^1 \leftrightarrow H_{f(\nu_{\text{SR}})}^2, \tag{3.9}$$

$$\alpha_{\text{num}} = \left( s_{\tau_0} H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) \left\{ \alpha_0 [H_{\nu_0-1}^2(x_0) - H_{\nu_0+1}^2(x_0)] + \beta_0 [H_{\nu_0-1}^1(x_0) - H_{\nu_0+1}^1(x_0)] \right\} + [\alpha_0 H_{\nu_0}^2(x_0) + \beta_0 H_{\nu_0}^1(x_0)] \left\{ \frac{\epsilon_{\text{SR}}}{\epsilon_0} [H_{\nu_{\text{SR}}-1}^1(x_{\text{SR}}) - H_{\nu_{\text{SR}}+1}^1(x_{\text{SR}})] + \frac{a_t H_t}{k} H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) \left[ -2 \left( 1 - \frac{\epsilon_{\text{SR}}}{\epsilon_0} \right) + s_{\tau_0} |\epsilon_0 - 1| + \frac{\epsilon_{\text{SR}}}{\epsilon_0} |\epsilon_{\text{SR}} - 1| \right] \right\} \right), \tag{3.10}$$

$$\alpha_{\text{denom}} = \left( \begin{array}{c} H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) [H_{\nu_{\text{SR}}+1}^2(x_{\text{SR}}) - H_{\nu_{\text{SR}}-1}^2(x_{\text{SR}})] \\ - H_{\nu_{\text{SR}}}^2(x_{\text{SR}}) [H_{\nu_{\text{SR}}+1}^1(x_{\text{SR}}) - H_{\nu_{\text{SR}}-1}^1(x_{\text{SR}})] \end{array} \right). \tag{3.11}$$

Here we have defined  $x_0 = |\epsilon_0 - 1|^{-1}(k/a_t H_t)$ ,  $x_{\text{SR}} = |\epsilon_{\text{SR}} - 1|^{-1}(k/a_t H_t)$  and  $s_{\tau_0} = \text{sign}(\tilde{\tau}_0)$ .

A special case of interest is a transition for which  $\alpha_0 = 1$  and  $\beta_0 = 0$ , corresponding to a transition from the state of lowest energy density prior to the transition. We will concentrate on this case for the remainder of the paper until section 5. The Bogoliubov parameters are oscillatory in nature, but in practice the oscillations can not be resolved experimentally and it is appropriate to approximate  $\beta_{f,k} \approx \beta_f$ .

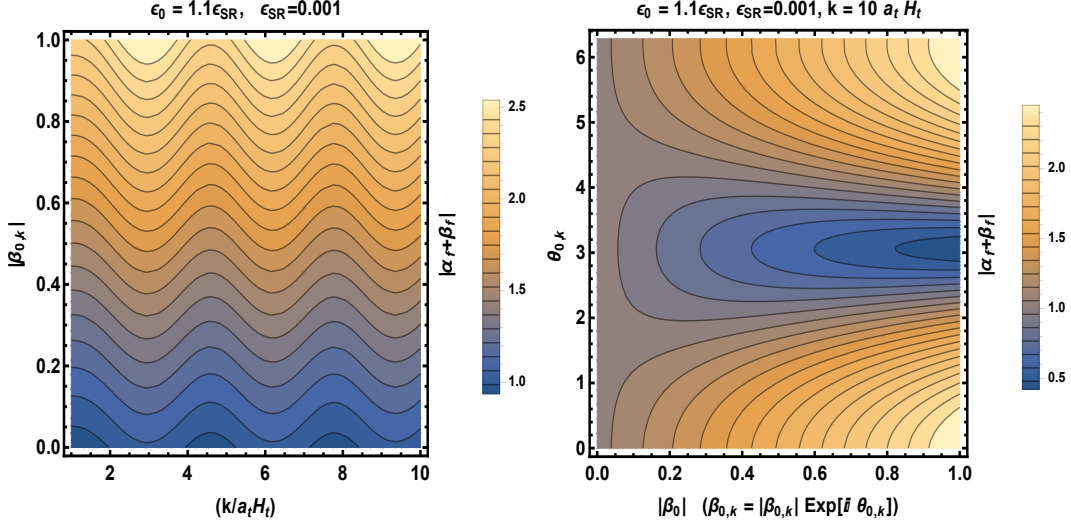
To clarify what we mean when we state that the oscillations cannot be resolved experimentally, we compare the scale of oscillations to the binning scale used by the Planck experiment [14]. The baseline Plik likelihood bin sizes are  $\Delta l = 1$  for  $l < 30$ ,  $\Delta l = 5$  for  $30 \leq l \leq 99$ ,  $\Delta l = 9$  for  $100 \leq l \leq 1503$ ,  $\Delta l = 17$  for  $1504 \leq l \leq 2013$  and  $\Delta l = 33$  for  $2014 \leq l \leq 2508$ . The oscillations in  $\alpha_{f,k}$  and  $\beta_{f,k}$  are controlled by  $k\tau_{0,t}$ , which results in several oscillations in a  $\Delta l = 1$  window.

Having computed the excitation spectrum that results from an instant transition, we can translate the bounds obtained in the previous section into bounds on  $w_0$ .

---

<sup>3</sup>Note that  $H_{\nu}^{1,2}(x \rightarrow \infty) \rightarrow e^{\pm ix}$ .

From our strongest bounds on  $|\beta_f|$  summarized in Table 3 and section 2.3.2, we may tabulate the largest allowed  $w_0$  for a given  $\epsilon_{\text{SR}}$ . Consider the effect of  $\beta_0 = |\beta_0| \exp(i\theta_0)$ . We see from Figure 2 that  $|\alpha_f + \beta_f|$  is maximal for  $\{|\beta_0|, \theta_0\} = \{\text{large}, 0\}$  and minimal for  $\{|\beta_0|, \theta_0\} = \{\text{large}, \pi\}$  for the parameter values specified on the plots. Since we have identified both an upper and lower bound on  $|\alpha_f + \beta_f|$ , we will take the intermediate case of  $\alpha_0 = 1$  and  $\beta_0 = 0$ . In Table 4 we present the bounds on the fractional change of  $w$  and  $\epsilon$ .



**Figure 2.** Amplitude of excitations  $|\alpha_{f,k} + \beta_{f,k}|$  after transition for initially excited state with  $\beta_{0,k} \neq 0$ . We have taken the form of the excitation prior to the transition to be  $\beta_{0,k} = |\beta_{0,k}| \exp(i\theta_{0,k})$  for arbitrary  $\theta_{0,k}$  in the right plot and  $\theta_{0,k} = 0$  in the left plot. These plots indicate that a larger  $|\beta_{0,k}|$  typically leads to a larger deviation of  $|\alpha_{f,k} + \beta_{f,k}|$  from unity. This justifies our choice of  $\beta_{0,k} = 0$  to derive our bounds presented in Table 4.

Observed Multipoles Excited	Relevant Bound	$100 \left  \frac{w_0 - w_{\text{SR}}}{w_{\text{SR}}} \right $	$100 \left  \frac{\epsilon_0 - \epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} \right $
$l \lesssim 30$	$0.71 \lesssim  \alpha_{f,k} + \beta_{f,k}  \lesssim 1.41$	$\lesssim 0.07$	$\lesssim 94$
$l > 30$ and lower	$0.97 <  \alpha_{f,k} + \beta_{f,k}  < 1.022$	$\lesssim 0.003$	$\lesssim 4.3$

**Table 4.** The maximal allowed  $\epsilon_0$  for a given  $\epsilon_{\text{SR}} < \epsilon_0$ . We have reported the bounds for  $\epsilon_{\text{SR}} = 10^{-3}$ , though we have confirmed that the bound on the fractional change of  $\epsilon$  is not numerically sensitive to this input. We have chosen  $\alpha_0 = 1$  and  $\beta_0 = 0$ .

If the transition to inflation is well approximated as an instantaneous transition with an initial  $w_0$  larger than is stated in Table 4, the modes which are excited cannot comprise our observable CMB. Note that modes which are super-Planckian at the

time of transition should be described by the Bunch-Davies vacuum when their momenta redshift to become sub-Planckian in order for the stress-energy tensor to be renormalizable [19].

We note for completeness that the bounds on the fractional change in  $\epsilon$  are more restrictive if we consider a transition in which  $\epsilon$  increases. The bounds are explicitly given in Table 5. In Figure 5 we demonstrate how the morphology of the spectrum changes depending on whether the step in  $\epsilon$  is to a smaller or larger  $\epsilon$ .

Observed Multipoles Excited	Relevant Bound	$100 \frac{w_0-w_{\text{SR}}}{w_{\text{SR}}} $	$100 \frac{\epsilon_0-\epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} $
$l \lesssim 30$	$0.71 \lesssim  \alpha_{f,k} + \beta_{f,k}  \lesssim 1.41$	$\lesssim 0.03$	$\lesssim 43$
$l > 30$ and lower	$0.97 <  \alpha_{f,k} + \beta_{f,k}  < 1.022$	$\lesssim 0.003$	$\lesssim 3.3$

**Table 5.** The maximal allowed  $\epsilon_0$  for a given  $\epsilon_{\text{SR}} > \epsilon_0$ . We have reported the bounds for  $\epsilon_{\text{SR}} = 10^{-3}$ , though we have confirmed that the bound on the fractional change of  $\epsilon$  is not numerically sensitive to this input. We have chosen  $\alpha_0 = 1$  and  $\beta_0 = 0$ .

### 3.3 Comparison with Previous Work

There have been many previous studies analyzing equation of state transitions [6, 7, 9, 19, 27–31]. Some recent examples with which we could easily compare our matching criteria are [6, 7, 28–30]. The matching conditions used in those studies do not agree with the matching conditions presented in equation (3.1). We also note that studies which numerically evolve the Mukhanov-Sasaki equation without making approximations for  $\{\dot{z}, \ddot{z}\}$  should yield the correct result if a proper step size is chosen so that the transition is sampled.

One of our conclusions is that only transitions from one inflationary phase to another are allowed to be imprinted on the observable CMB. A special case which has been studied previously is steps in the inflationary potential which are modeled by a hyperbolic tangent of the field value [31]. We find that even for the most violent case of an instant transition with a step size  $|\Delta V|/V_0 = \epsilon_0/3$ , the fractional change in  $\epsilon$  almost satisfies our least restrictive bound presented in Table 5. To see this explicitly, note that the initial kinetic energy is given by  $K_0 = |\Delta V|$  and therefore  $K_f = 2K_0$  by energy conservation. The fractional change in  $\epsilon \approx 3K/V$  is given by

$$\epsilon_0 = \frac{1}{2} \frac{\epsilon_{\text{SR}}}{(1 + \epsilon_{\text{SR}}/6)} \approx \frac{1}{2} \epsilon_{\text{SR}}, \quad \left| \frac{\epsilon_0 - \epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} \right| \approx 0.5. \quad (3.12)$$

## 4 Excitation Mechanism: Gradual Transition

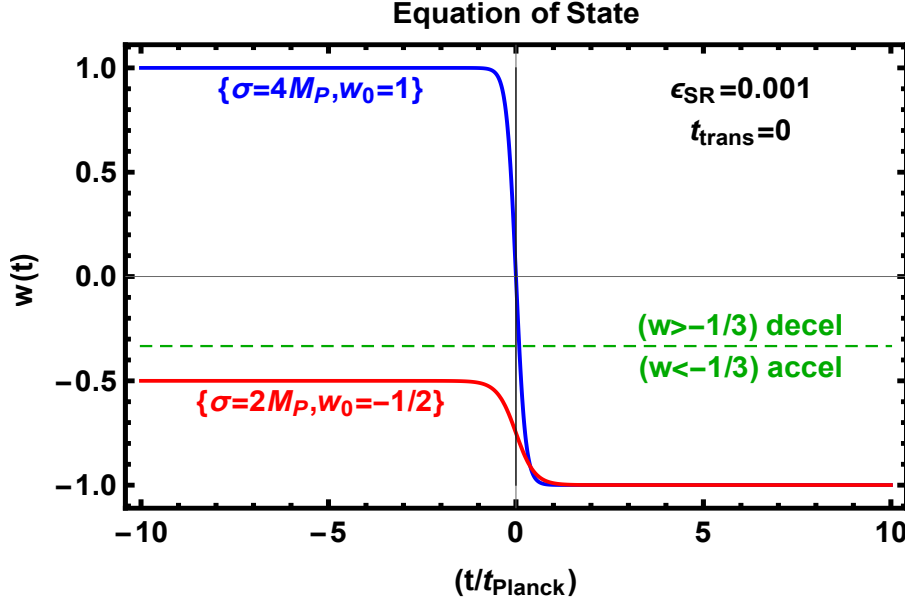
### 4.1 Transition Model

Our parameterization is of the form

$$w(t) = w_0 + \frac{1}{2}(w_{\text{SR}} - w_0) (1 + \tanh [\sigma(\Delta t)_t]) , \quad w_{\text{SR}} = -1 + \frac{2}{3}\epsilon_{\text{SR}}. \quad (4.1)$$

Figure 3 illustrates the transition for two different values of  $w_0$  and  $\sigma$ . The slow-roll parameter is explicitly given by

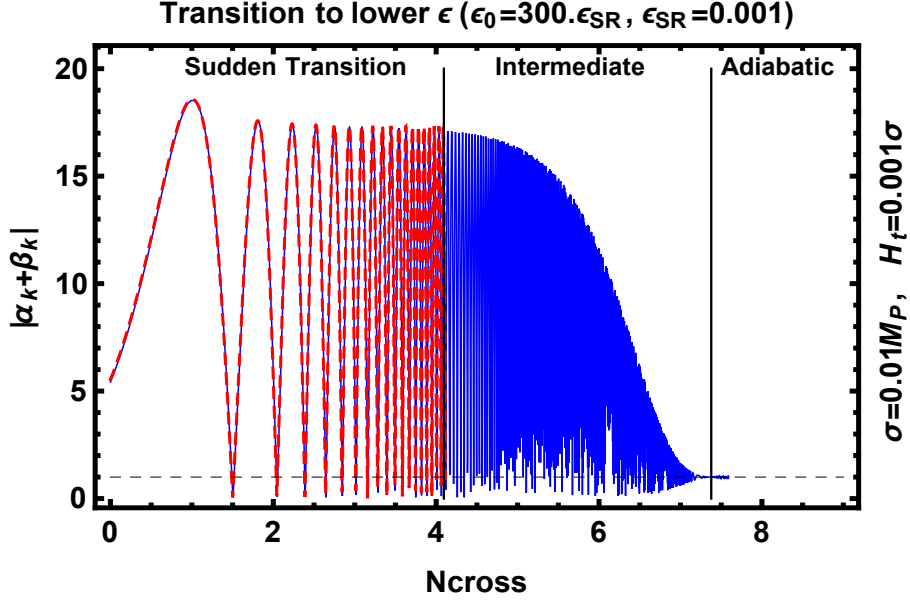
$$\epsilon(t) = \frac{3}{2}(1 + w) = \frac{1}{2}(\epsilon_{\text{SR}} + \epsilon_0 + (\epsilon_{\text{SR}} - \epsilon_0) \tanh [\sigma(\Delta t)_t]) . \quad (4.2)$$



**Figure 3.** The equation of state as a function of the cosmic time for gradual transitions parameterized by (4.1). Though not explicitly plotted, we also have studied transitions which increase the equation of state parameter (see Figure 5).

### 4.2 Observables: Allowed Parameter Space

The gradual transition has three cases for modes depending on whether a mode experiences the transition as sudden, adiabatic or an intermediate case between the two. For an example of these three regimes, see Figure 4. We have explicitly compared the spectrum morphology for a case of transitioning to a lower  $\epsilon$  to the case of transitioning to a higher  $\epsilon$  in Figure 5.



**Figure 4.** A comparison of the spectrum enhancement obtained from the numerical solution (solid blue line) and the analytical formula (dashed red line) obtained from equation (3.9). The parameter  $N_{\text{cross}} = \log(k/a_t H_t)$  is the number of efolds from the time of the transition until the mode exits the horizon. For the sake of clarity, we end the analytically obtained line at 4 efolds. The numerical solution stops at slightly over 7.5 efolds. The separators between the three regimes of modes are approximately placed. We have chosen  $\sigma/H = 10^3$  for illustration, though as  $\sigma/H$  approaches unity fewer sub-horizon modes are excited. For  $\sigma/H \sim 1$ , there are still more than two efolds of excited modes which were sub-horizon at the time of transition.

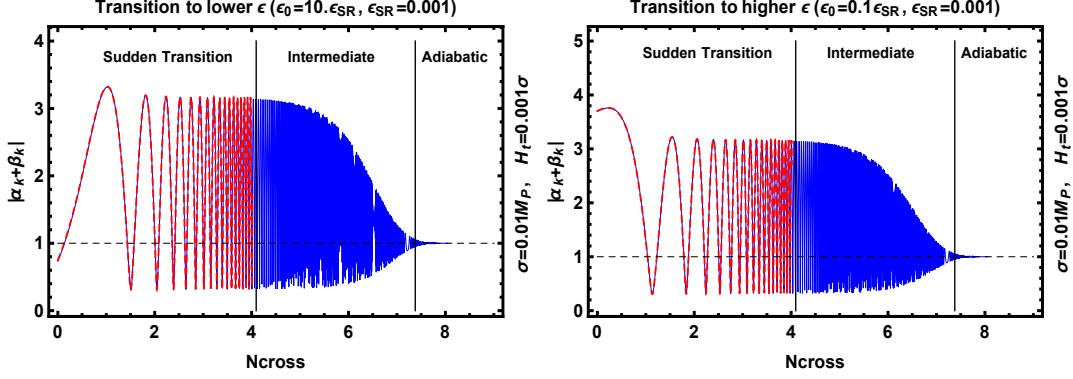
Comparing the  $\dot{\epsilon}$  friction term and the frequency term in the equation of motion (2.7) provides an intuition for the three cases. This is most easily done by rescaling the curvature perturbation in order to eliminate the friction term altogether. The appropriate rescaling is given by

$$\mathcal{R}_k = (a^3 \epsilon)^{-1/2} \bar{\mathcal{R}}_k. \quad (4.3)$$

The resulting rescaled equation of motion is given by

$$\ddot{\bar{\mathcal{R}}}_k + \bar{\omega}_k^2 \bar{\mathcal{R}}_k = 0, \quad \bar{\omega}_k^2 = \left[ \left( \frac{k}{a} \right)^2 - \left( \frac{9}{4} H^2 + \frac{3}{2} H \frac{\dot{\epsilon}}{\epsilon} - \frac{1}{4} \frac{\dot{\epsilon}^2}{\epsilon^2} \right) - \frac{1}{2} \left( 3\dot{H} + \frac{\ddot{\epsilon}}{\epsilon} \right) \right]. \quad (4.4)$$

Modes for which the term proportional to  $k$  dominates the effective frequency  $\bar{\omega}_k$  tend to be adiabatic since they satisfy  $|\dot{\bar{\omega}}_k/\bar{\omega}_k^2| \ll 1$  during the transition. Likewise, modes



**Figure 5.** Spectrum morphology comparison for a transition to a higher  $\epsilon$  versus a transition to a lower  $\epsilon$ . Note that a transition with  $\epsilon_0 > \epsilon_{\text{SR}}$  tends to reach its first peak for higher momenta than the case of  $\epsilon_0 < \epsilon_{\text{SR}}$ . The separators between the three regimes of modes are approximately placed.

Condition	Cases
$ \dot{\bar{\omega}}_k / \bar{\omega}_k^2  \gg 1$	Sudden Transition
$ \dot{\bar{\omega}}_k / \bar{\omega}_k^2  \ll 1$	Intermediate Modes
	Adiabatic

**Table 6.** Three cases for a given momentum mode depending on how deep inside of the horizon it is during the transition.

which satisfy  $|\dot{\bar{\omega}}_k / \bar{\omega}_k^2| \gg 1$  during the transition tend to experience a sudden transition. We summarize these behaviors in Table 6.

From Figure 4 there are clearly two distinct cases in which we may observe excited modes:

1. We observe sudden/intermediate modes with an amplitude close to the maximum amplitude, in which case the fractional change in  $\epsilon$  is strongly bounded (see Table 4).
2. We observe only intermediate modes at the low amplitude tail of the spectrum, in which case the fractional change in  $\epsilon$  may have been large but the modes with a large amplitude are hidden outside of our horizon.

The second case may allow for large fractional change in  $\epsilon$  compared to the bounds presented in Table 4, but it requires fine tuning to ensure that the large amplitude modes are not observed. The fine tuning becomes more concerning as the fractional change in  $\epsilon$  increases, because difference in amplitude between the hidden modes and the visible modes increases dramatically. Moreover, it is not clear we would be able to infer  $\epsilon_0$  from the observation of the low amplitude tail modes.

## 5 Implications for Low Multipole Scalar Power Spectrum Suppression

Observationally there is a suppression of power for multipoles of  $20 \lesssim l \lesssim 30$  [14]. A brief discussion of the history associated with discovering and modeling this anomaly is contained in [6]. The observed power suppression is approximately given by

$$\frac{\Delta_{\mathcal{R}}^2|_{\text{expected } 20 < l < 30}}{\Delta_{\mathcal{R}}^2|_{\text{actual } 20 < l < 30}} \approx \frac{600}{1000} = 60\%. \quad (5.1)$$

To suppress the scalar power spectrum on large scales, one would need the scale dependence of  $|\alpha_k + \beta_k|$  to suppress power for the relevant multipoles.

Based on our discussion in the previous section, there are two possibilities:

1. Sudden transition modes comprise the entire CMB, in which case the bounds from Table 4 hold.
2. Only the lowest  $l$  modes are excited.

For the first case the largest relative suppression that may be obtained is

$$\frac{|\alpha_k + \beta_k|_{LB}^2}{|\alpha_k + \beta_k|_{UB}^2} \approx \left( \frac{0.97}{1.022} \right)^2 = 90\%, \quad (5.2)$$

This is an insufficient amount of suppression.

For the second case we note that the envelope of the excited spectrum typically monotonically decays from a large excitation amplitude to a smaller excitation amplitude as is depicted in Figure 4. Since the modes between  $l \approx 10$  and  $l \approx 20$  do not show a power suppression to the same extent that the  $20 \lesssim l \lesssim 30$  modes do, we do not think that the transitions which we have studied are good candidates for explaining the power suppression anomaly. It may be possible to finely tune the pre-transition excitation parameter  $\beta_{0,k}$  (see Figure 2) to obtain the desired spectrum [32], but it is not obvious what mechanism could give rise to such a selected excitation.

## 6 Conclusions

We have revisited the physics of early universe transitions to slow-roll inflation. The proper matching conditions must be used when determining the spectrum of excited fluctuations across an equation of state transition. A careful numerical study of the problem agrees with matching  $\{\mathcal{R}, \epsilon \dot{\mathcal{R}}\}$  as opposed to  $\{\mathcal{R}, \dot{\mathcal{R}}\}$ . There are three regimes present in a gradual transition: modes which experience a sudden transition, adiabatic



modes and modes which interpolate between those two regimes which we call intermediate modes.

If the modes comprising the visible CMB contain imprints of the transition, we have shown that the pre-transition universe must likewise be an inflationary period. The only exception is if the cosmic variance limited modes are comprised of intermediate modes generated by a transition from a large  $w_0$ . We have also argued that it is very unlikely that equation of state transitions can explain the low multipole power suppression observed in the CMB since it requires a very localized excitation in momentum space prior to the transition.

Our results state that the physics which preceded inflation is not likely to be imprinted on the observable CMB. This is a discouraging result from the perspective of using CMB observations to gain insight into the earliest stages of our universe. However, it is encouraging since it allows us to interpret cosmological observations in the context of inflationary cosmology without having to worry about potential ambiguities introduced by pre-inflationary physics.

## Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant Number PHY-1316033 and Grant Number PHY-1521186.

## References

- [1] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2015 results. XX. Constraints on inflation,” [arXiv:1502.02114 \[astro-ph.CO\]](#).
- [2] D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka, “Open inflation in the landscape,” *Phys. Rev. D* **84**, 043513 (2011) doi:10.1103/PhysRevD.84.043513 [[arXiv:1105.2674 \[hep-th\]](#)].
- [3] W. E. East, M. Kleban, A. Linde and L. Senatore, “Beginning inflation in an inhomogeneous universe,” [arXiv:1511.05143 \[hep-th\]](#).
- [4] M. Kleban and L. Senatore, “Inhomogeneous Anisotropic Cosmology,” [arXiv:1602.03520 \[hep-th\]](#).
- [5] D. J. Schwarz and E. Ramirez, “Just enough inflation,” doi:10.1142/9789814374552\_0180 [arXiv:0912.4348 \[hep-ph\]](#).
- [6] M. Cicoli, S. Downes, B. Dutta, F. G. Pedro and A. Westphal, “Just enough inflation: power spectrum modifications at large scales,” *JCAP* **1412**, no. 12, 030 (2014) doi:10.1088/1475-7516/2014/12/030 [[arXiv:1407.1048 \[hep-th\]](#)].
- [7] C. R. Contaldi, M. Peloso, L. Kofman and A. D. Linde, “Suppressing the lower multipoles in the CMB anisotropies,” *JCAP* **0307**, 002 (2003) doi:10.1088/1475-7516/2003/07/002 [[astro-ph/0303636](#)].
- [8] B. R. Greene, K. Schalm, G. Shiu and J. P. van der Schaar, “Decoupling in an expanding universe: Backreaction barely constrains short distance effects in the CMB,” *JCAP* **0502**, 001 (2005) doi:10.1088/1475-7516/2005/02/001 [[hep-th/0411217](#)].
- [9] A. Aravind, D. Lorshbough and S. Paban, “Non-Gaussianity from Excited Initial Inflationary States,” *JHEP* **1307**, 076 (2013) doi:10.1007/JHEP07(2013)076 [[arXiv:1303.1440 \[hep-th\]](#)].
- [10] R. Flauger, D. Green and R. A. Porto, “On squeezed limits in single-field inflation. Part I,” *JCAP* **1308**, 032 (2013) doi:10.1088/1475-7516/2013/08/032, 10.1088/1475-7516/2013/08/032/ [[arXiv:1303.1430 \[hep-th\]](#)].
- [11] A. Aravind, D. Lorshbough and S. Paban, “Bogoliubov Excited States and the Lyth Bound,” *JCAP* **1408**, 058 (2014) doi:10.1088/1475-7516/2014/08/058 [[arXiv:1403.6216 \[astro-ph.CO\]](#)].
- [12] K. A. Malik and D. Wands, “Cosmological perturbations,” *Phys. Rept.* **475**, 1 (2009) doi:10.1016/j.physrep.2009.03.001 [[arXiv:0809.4944 \[astro-ph\]](#)].
- [13] R. Adam *et al.* [Planck Collaboration], “Planck 2015 results. I. Overview of products and scientific results,” [arXiv:1502.01582 \[astro-ph.CO\]](#).

- [14] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2015 results. XIII. Cosmological parameters,” [arXiv:1502.01589 \[astro-ph.CO\]](#).
- [15] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2015 results. XVII. Constraints on primordial non-Gaussianity,” [arXiv:1502.01592 \[astro-ph.CO\]](#).
- [16] H. V. Peiris and L. Verde, “The Shape of the Primordial Power Spectrum: A Last Stand Before Planck,” *Phys. Rev. D* **81**, 021302 (2010) doi:10.1103/PhysRevD.81.021302 [[arXiv:0912.0268 \[astro-ph.CO\]](#)].
- [17] P. Hunt and S. Sarkar, “Reconstruction of the primordial power spectrum of curvature perturbations using multiple data sets,” *JCAP* **1401**, 025 (2014) doi:10.1088/1475-7516/2014/01/025 [[arXiv:1308.2317 \[astro-ph.CO\]](#)].
- [18] P. Hunt and S. Sarkar, “Search for features in the spectrum of primordial perturbations using Planck and other datasets,” *JCAP* **1512**, no. 12, 052 (2015) doi:10.1088/1475-7516/2015/12/052 [[arXiv:1510.03338 \[astro-ph.CO\]](#)].
- [19] D. Carney, W. Fischler, S. Paban and N. Sivanandam, “The Inflationary Wavefunction and its Initial Conditions,” *JCAP* **1212**, 012 (2012) doi:10.1088/1475-7516/2012/12/012 [[arXiv:1109.6566 \[hep-th\]](#)].
- [20] R. Holman and A. J. Tolley, “Enhanced Non-Gaussianity from Excited Initial States,” *JCAP* **0805**, 001 (2008) doi:10.1088/1475-7516/2008/05/001 [[arXiv:0710.1302 \[hep-th\]](#)].
- [21] J. Ganc, “Calculating the local-type fNL for slow-roll inflation with a non-vacuum initial state,” *Phys. Rev. D* **84**, 063514 (2011) doi:10.1103/PhysRevD.84.063514 [[arXiv:1104.0244 \[astro-ph.CO\]](#)].
- [22] A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari and G. Shiu, “Reconciliation of High Energy Scale Models of Inflation with Planck,” *JCAP* **1402**, 025 (2014) doi:10.1088/1475-7516/2014/02/025 [[arXiv:1306.4914 \[hep-th\]](#)].
- [23] A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari and G. Shiu, “Non-BunchDavis initial state reconciles chaotic models with BICEP and Planck,” *Phys. Lett. B* **737**, 98 (2014) doi:10.1016/j.physletb.2014.08.038 [[arXiv:1403.6099 \[hep-th\]](#)].
- [24] S. Kundu, “Inflation with General Initial Conditions for Scalar Perturbations,” *JCAP* **1202**, 005 (2012) doi:10.1088/1475-7516/2012/02/005 [[arXiv:1110.4688 \[astro-ph.CO\]](#)].
- [25] S. Kundu, “Non-Gaussianity Consistency Relations, Initial States and Back-reaction,” *JCAP* **1404**, 016 (2014) doi:10.1088/1475-7516/2014/04/016 [[arXiv:1311.1575 \[astro-ph.CO\]](#)].
- [26] N. Deruelle and V. F. Mukhanov, “On matching conditions for cosmological perturbations,” *Phys. Rev. D* **52**, 5549 (1995) doi:10.1103/PhysRevD.52.5549 [[gr-qc/9503050](#)].

- [27] R. K. Jain, P. Chingangbam, J. O. Gong, L. Sriramkumar and T. Souradeep, “Punctuated inflation and the low CMB multipoles,” JCAP **0901**, 009 (2009) doi:10.1088/1475-7516/2009/01/009 [[arXiv:0809.3915](#) [[astro-ph](#)]].
- [28] P. Chen and Y. H. Lin, “What initial condition of inflation would suppress the large-scale CMB spectrum?,” Phys. Rev. D **93**, no. 2, 023503 (2016) doi:10.1103/PhysRevD.93.023503 [[arXiv:1505.05980](#) [[gr-qc](#)]].
- [29] S. Das, G. Goswami, J. Prasad and R. Rangarajan, “Revisiting a pre-inflationary radiation era and its effect on the CMB power spectrum,” JCAP **1506**, no. 06, 001 (2015) doi:10.1088/1475-7516/2015/06/001 [[arXiv:1412.7093](#) [[astro-ph.CO](#)]].
- [30] Y. Cai, Y. T. Wang and Y. S. Piao, “Preinflationary primordial perturbations,” Phys. Rev. D **92**, no. 2, 023518 (2015) doi:10.1103/PhysRevD.92.023518 [[arXiv:1501.01730](#) [[astro-ph.CO](#)]].
- [31] P. Adshead, C. Dvorkin, W. Hu and E. A. Lim, “Non-Gaussianity from Step Features in the Inflationary Potential,” Phys. Rev. D **85**, 023531 (2012) doi:10.1103/PhysRevD.85.023531 [[arXiv:1110.3050](#) [[astro-ph.CO](#)]].
- [32] L. Sriramkumar and T. Padmanabhan, “Initial state of matter fields and trans-Planckian physics: Can CMB observations disentangle the two?,” Phys. Rev. D **71**, 103512 (2005) doi:10.1103/PhysRevD.71.103512 [[gr-qc/0408034](#)].